

How much energy do we spend in a 10-minute shower?

Authors

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Type of activity

An exercise is proposed in which the cost of a shower must be calculated based on fuel consumption. The expense is related to the flow rate and shower time, which, although it allows delving into thermodynamic concepts, sensitizes, and raises awareness among students about energy savings. SDG 12 "Ensure sustainable consumption and production patterns".

Target educational level

The complete problem can be requested from university students of Physics, Chemistry and Engineering.

The problem is solved in different phases, so it can be adapted to high school or secondary school levels if the results of the first phases are provided as data of the problem.



Problem statement

We want to heat the water needed for a shower from 10°C to 30°C. The combustion boiler works with natural gas, methane, at 25°C and 1 bar and with an efficiency of 60%.

The methane is burned with 100% of the theoretical air, which enters the chamber at the same pressure and temperature as the fuel and with a relative humidity of 40%.

The combustion products leave the boiler at a temperature of 800K, with a sensible enthalpy of 138 MJ/kmol CH₄.



If the water flow with the tap fully open is 7 L/minute and the shower lasts 10 minutes,

- Calculate the price in euros of the shower, knowing that the cost of methane is €1.2 per m³.
- Estimate the economic savings per shower that would mean lowering the water flow to 3.5 L/min.
- Estimate the economic savings per shower that would result from lowering the water flow to 3.5 L/min and if the shower time is also reduced to 5 minutes?

Data: $\Delta H_f(\text{CH}_4) = -74850 \text{ kJ/kmol}$

$\Delta H_f(\text{CO}_2) = -393520 \text{ kJ/kmol}$

$P_{\text{v H}_2\text{O}}(25^\circ\text{C}) = 0.03169 \text{ bar}$

$c_p(\text{H}_2\text{O}, \text{l}) = 4.18 \text{ kJ/kg} \cdot \text{K}$

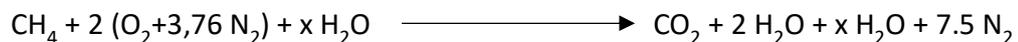
$\Delta H_f(\text{H}_2\text{O}) = -241820 \text{ kJ/kmol}$

$\Delta H_s(\text{products}) = 138 \text{ MJ/kmol}$

$v_{\text{CH}_4}(25^\circ\text{C}, 1 \text{ bar}) = 1.5 \text{ m}^3/\text{kg}$

Problem solution:

- Adjusted combustion reaction:



- Determination of the water present in the air (humidity):

$$\phi = \frac{P_{\text{H}_2\text{O}}}{P_{\text{VH}_2\text{O}}(25^\circ\text{C})} = \frac{Y_{\text{H}_2\text{O}} P_T}{P_{\text{VH}_2\text{O}}(25^\circ\text{C})}$$

$$Y_{\text{H}_2\text{O}} = 0,4 * 0,03169 = 0,01268$$

$$Y_{\text{H}_2\text{O}} = \frac{n_{\text{H}_2\text{O}}}{n_{\text{H}_2\text{O}} + n_{\text{aire}}} = \frac{n_{\text{H}_2\text{O}}}{n_{\text{H}_2\text{O}} + 2 * 4,76}$$

$$n_{\text{H}_2\text{O}} = 0,1222$$

- Combustion reaction adjusted with air humidity:



- Determination of the enthalpy of combustion:

$$\Delta H_C = \Delta H_{\text{formation}} + \Delta H_{\text{sensible}}$$



$$\Delta H_{formation} = 1 * \Delta H_f(CO_2) + 2 * \Delta H_f(H_2O) - \Delta H_f(CH_4) = -802310 \text{ kJ/kmol}$$

$$\Delta H_C = -802310 + 138000 = -663810 \text{ kJ/kmol} = -41488 \text{ kJ/kg}$$

- Determination of fuel consumption to heat water based on flow rate (kg water/min) and shower time (min).

$$\Delta H_{combustion} * (m_{CH_4}) = \Delta H_{water} * (m_{water})$$

$$\Delta H_{water} = 4,18 * (30-10) = 83.6 \text{ kJ/kg water}$$

$$41488 \text{ (kJ/kg}_{methane}) * 0.6 * m_{methane}(\text{kg}) = 83.6 \text{ (kJ/kg}_{water}) * m_{water} \text{ (kg}_{water}/\text{min}) * t(\text{min})$$

$$m_{methane} \text{ (kg)} = 3.36 \cdot 10^{-3} * m_{water} \text{ (kg}_{water}/\text{min}) * t(\text{min})$$

$$V_{methane} \text{ (m}^3) = 3.36 \cdot 10^{-3} \text{ kg} * 1.5 \text{ (m}^3/\text{kg}) * m_{water} \text{ (kg}_{water}/\text{min}) * t(\text{min})$$

$$\text{Shower cost} = 5.04 \cdot 10^{-3} \text{ m}^3_{methane} * m_{water} \text{ (kg}_{water}/\text{min}) * t(\text{min}) * 1.2 \text{ (€/m}^3)$$

- Shower cost from 10 min to 7 kg of water/min: 0.42 €
- Shower cost of 10 min at 3.5 Kg water/min: 0.21 €
- Shower cost from 5 min to 7 kg of water/min: 0.21 €
- Shower cost from 5 min to 3.5 kg water/min: 0.11 €