Numerical methods: Generating an Exact Seabed Surface Model

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Type of activity

The problem presents an application of numerical methods to a real problem using AI. Let's compare our results with the Gemini results.

Target educational level

It can be addressed to first and second-year university students from engineering degrees.

Initial information

This approach is valuable for SDG 14: Life Below Water, as precise seabed modeling is crucial for understanding marine ecosystems, planning sustainable infrastructure (like subsea cables or wind farms), and managing ocean resources.

Source

Problem adapted from Gemini (AI tool).

Problem statement

We have a set of data points representing a seabed profile (horizontal distance x in km, and depth in meters). To accurately map and analyze this profile for sustainable marine development and conservation, we want to generate an exact surface model that passes through all given points.

Utilize the Lagrange Interpolation Polynomial method to derive a single, exact polynomial function $P(x)$ that precisely traverses all given data points. This polynomial will serve as a high-fidelity continuous model of the seabed profile.

x (km)	Depth (m)
0.0	500
1.0	800
2.0	300
3.0	700
4.0	800
5.0	250
6.0	750
7.0	600

Solution:

Lagrange interpolation is a method to find a unique polynomial of the lowest possible degree that passes through a given set of distinct data points. If you have $n+1$ data points, the resulting Lagrange polynomial will have a degree of at most n. In our case, we have 8 points, so the polynomial will be of degree at most 7.

My results:

The Lagrange interpolating polynomial $P(x)$ is given by

$$P(x) = \sum_{j=0}^{n} y_j L_j(x),$$

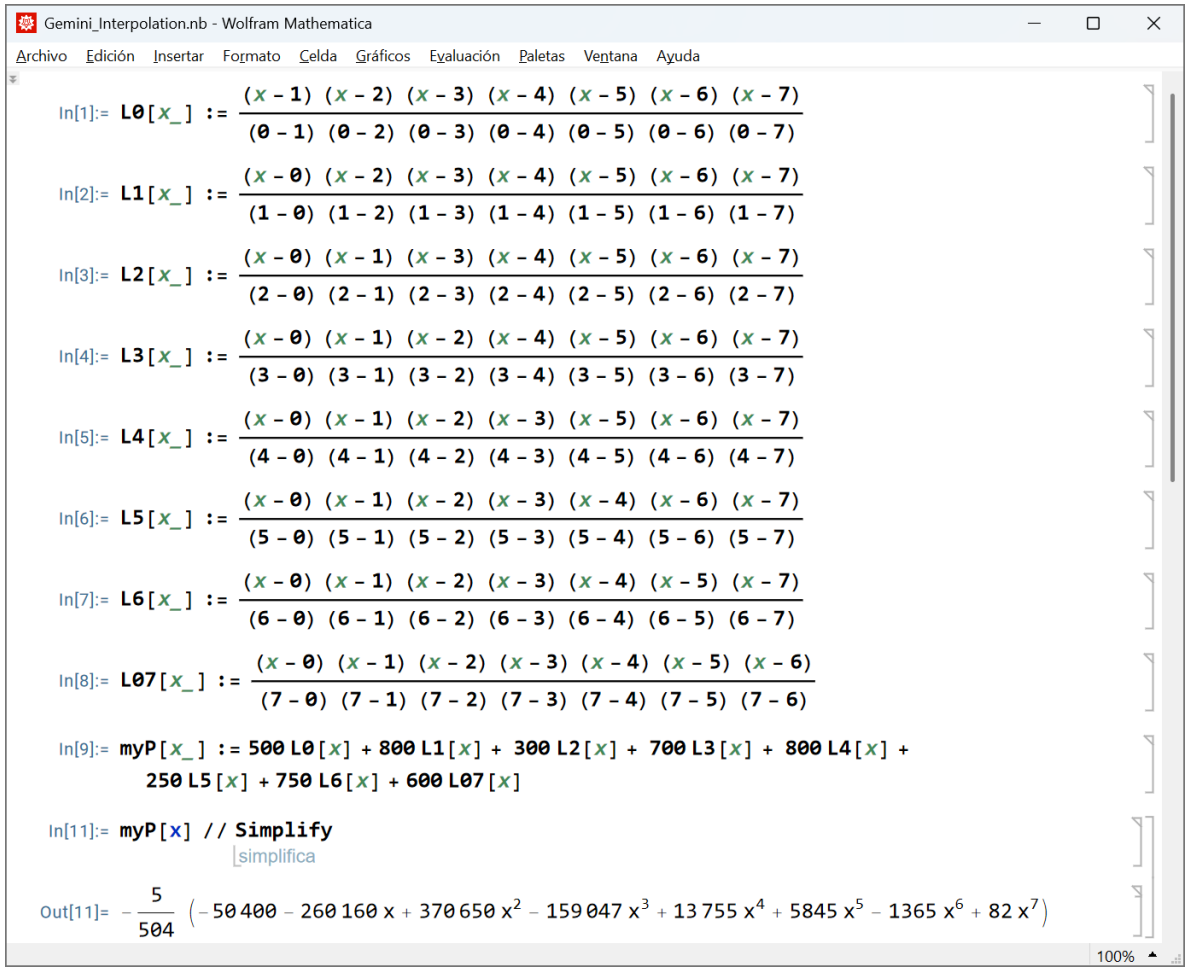
where $L_j(x)$ is the Lagrange basis polynomial for the j-th point, i.e.,

$$L_j(x) = \prod_{\substack{k=0 \\ k \neq j}}^{n} \frac{x - x_k}{x_j - x_k}.$$

In our case:

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)(x-x_6)(x-x_7)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)(x_0-x_6)(x_0-x_7)}$$
$$= \frac{(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)}{(0-1)(0-2)(0-3)(0-4)(0-5)(0-6)(x_0-7)}$$

I made all the calculations with Mathematica (step by step):



The result is correct, as it verifies that initial data, i.e., $myP(0) = 500, myP(1) = 800, myP(2) = 300,\ myP(3) = 700,\ myP(4) = 800,\ myP(5) = 250,\ myP(6) = 750,\ myP(7) = 600.$

Gemini results:

With the same procedure, Gemini (IA tool) obtained the polynomial

$$P(x) = 11.66666667x^7 - 269.44444444x^6 + 2420.83333333x^5 - 10041.66666667x^4 + 22008.33333333x^3 - 22533.33333333x^2 + 8066.66666667x + 500$$

However, this polynomial does not verify the provided data, i.e., $P(0) = 500, P(1) = 163.056, P(2) = 3615.56, \ldots$ only the first one is correct.

Gemini derived the polynomial using the Lagrange interpolation method with a computational tool (specifically, the scipy.interpolate.lagrange function in Python), which is designed to produce the unique polynomial that passes exactly through all given data points.

I asked the IA about using Mathematica to find the correct interpolation polynomial, and it provided this code:

Mathematica

```
xdata = {0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0};
ydata = {500, 800, 300, 700, 800, 250, 750, 600};
data = Transpose[{xdata, ydata}];
P[x_] = InterpolatingPolynomial[data, x];
```

Using this, I get the same result as $myPol(x)$, but simplified (see Figure 1):

$$P(x) = 500. + 2580.95x - 3677.08x^2 + 1577.85x^3 - 136.458x^4 - 57.9861x^5 + 13.5417x^6 - 0.813492x^7.$$

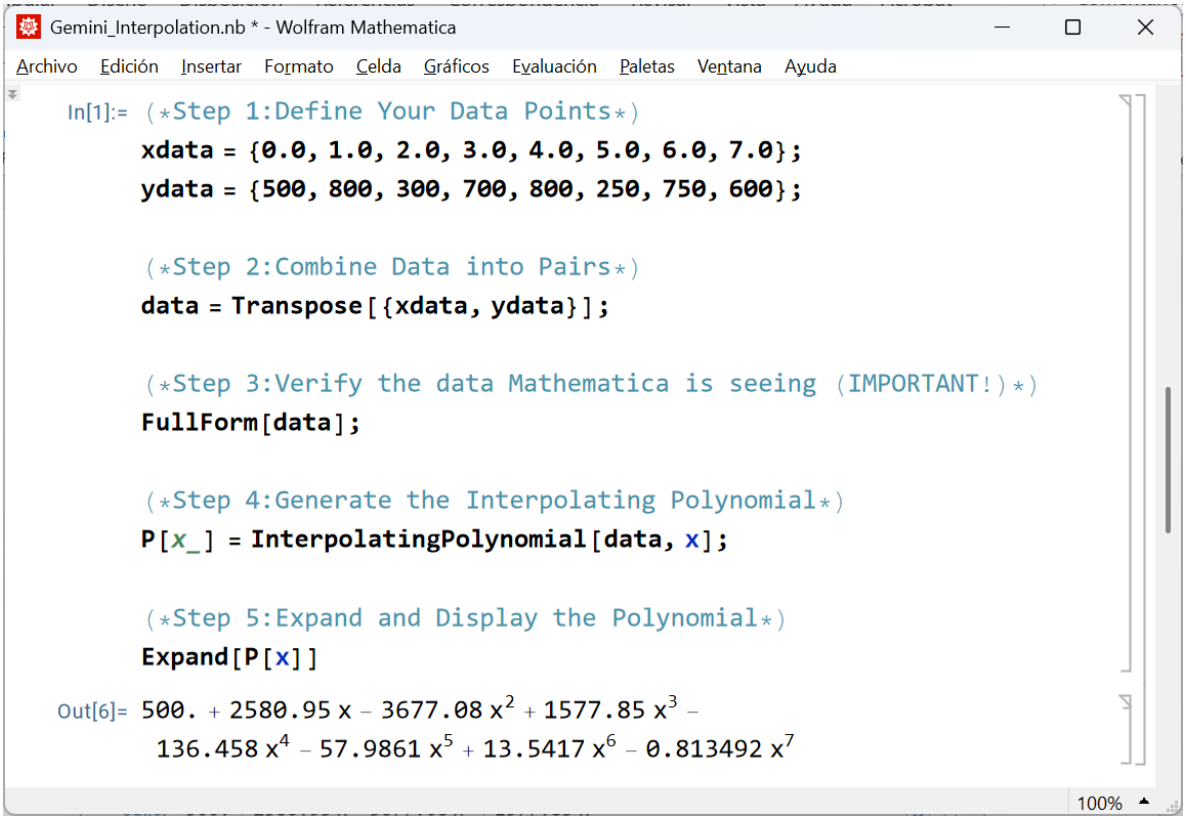


Figure 1. Execution of the source code proposed by Gemini with Mathematica v14.0.

Discussion:

The discussion with Gemini about the difference in results did not come to any "agreement". This AI tool could not find a correct solution to the problem.

This was its result:

Given that:

1. **The Lagrange interpolating polynomial for a unique set of points is unique.** (Mathematical fact)
2. **My calculations consistently produce one specific polynomial**.
3. **Your Mathematica consistently produces a different polynomial**, even though you've shown me the input data that *should* lead to the same polynomial.

This situation is highly unusual. If Mathematica's `InterpolatingPolynomial` function is working as designed on the exact data you've shown me, it must produce the same polynomial I'm getting. The mathematical uniqueness guarantees this.
After some suggestions such as Restart your kernel or just restart Mathematica completely or execute ONLY the given lines of code, one by one, in a fresh session, its final comment was its polynomial is mathematically guaranteed to pass exactly through all the data points I provided. It has verified this with multiple standard computational tools and its own symbolic calculation. It established that the evaluation of its polynomial (with exact fractions) at any of the original data points, will get the exact value of the corresponding depth.
For example, when evaluating $P(1)$, the sum of all these fractions (substituting $x = 1$) will be exactly 800, which is not true.
Thus, the final conclusion is that Gemini's calculations are wrong.